Comment on "Quantitative Condition is Necessary in Guaranteeing the Validity of the Adiabatic Approximation"

Recently, the authors of Ref.[1] claimed that they have proven the traditional adiabatic condition is a necessary condition. Here, it is claimed that there are some mistakes and an artificial over-strong constraint in [1], making its result inconvincible.

In their proof in [1], the author underestimated the contributions of other small components in a general adiabatic evolution, which is implied in Eq.(7) in [1]. A detailed version of Eq.(7) in [1] is as follows

$$|\psi(t)\rangle = \beta(t)|\psi^{adi}(t)\rangle + \delta(t)|\psi^{adi^{\perp}}(t)\rangle, \tag{1}$$

where $|\delta(t)| \ll 1$ and $\beta(t)^2 + |\delta(t)|^2 = 1$. In the following, time parameter t is omitted for convenience. Actually, Eq.(1) is equivalent to Eq.(10.54) in [2]. Differentiating both sides of Eq.(1) we get

$$|\dot{\psi}\rangle = \beta|\dot{\psi}^{adi}\rangle + \dot{\beta}|\psi^{adi}\rangle + \dot{\delta}|\psi^{adi^{\perp}}\rangle + \delta|\dot{\psi}^{adi^{\perp}}\rangle. \tag{2}$$

Substituting Eq.(1) into the Schrodinger's equation, and combining the result with Eq.(2), we get($\hbar = 1$)

$$\begin{aligned} i|\dot{\psi}^{adi}\rangle &= H|\psi^{adi}\rangle - i\frac{\dot{\beta}}{\beta}|\psi^{adi}\rangle + \frac{1}{\beta}(H\delta|\psi^{adi^{\perp}}\rangle \\ &\quad - i\dot{\delta}|\psi^{adi^{\perp}}\rangle - i\delta|\dot{\psi}^{adi^{\perp}}\rangle). \end{aligned} \tag{3}$$

As $\dot{\beta} = -\frac{|\delta|}{\beta}\frac{d|\delta|}{dt}$, a term which contains $\dot{\beta}$ is subleading in the right side of Eq.(3). To simplify the following discussion we may neglect its effect. So Eq.(6) in [1] declared to be a necessary condition is valid if and only if $\left\|\frac{1}{\beta}(-i\dot{\delta}|\psi^{adi^{\perp}}\rangle - i\delta|\dot{\psi}^{adi^{\perp}}\rangle)\right\|$ is small comparing to $\left\|H|\psi^{adi}\rangle\right\|$. Here and thereafter the symbol $\|\cdot\|$ denotes the norm of vector. This generally requires $\left|\dot{\delta}\right| \ll E_m$ and $\left\|\delta|\dot{\psi}^{adi^{\perp}}\rangle\right\| \ll E_m$ which are extra requirements in addition to the existing requirement of Eq.(7) in [1].

It's explicit that the 'Proof' in [1] can not be completed without the extra limitation Eq.(6) in [1]. Substituting Eq.(1) and Eq.(2) into Eq.(12) in [1] and neglect the term containing $\dot{\beta}$, we get

$$c_m = \frac{1}{E_m - E_n}[i\beta\gamma_1 + (i\dot{\delta} - E_n\delta)\gamma_2 + i\delta\gamma_3)], \tag{4}$$

where $\gamma_1 = \langle E_m|\dot{\psi}^{adi}\rangle$,$\gamma_2 = \langle E_m|\psi^{adi^{\perp}}\rangle$,$\gamma_3 = \langle E_m|\dot{\psi}^{adi^{\perp}}\rangle$. Terms containing δ can be ignored assuming δ is small enough,

$$c_m = \frac{1}{E_m - E_n}(i\beta\gamma_1 + i\dot{\delta}\gamma_2), \tag{5}$$

but terms containing $\dot{\delta}$ cannot be ignored [2]. Comparing Eq.(5) here with Eq.(13) in [1], one can see that the proof in [1] can only be applied to special systems. It is quite possible that neither of the two terms in the right side of Eq.(5) is small but their summation is small. In this case, quantitative condition is unnecessary. Specifically, the second example in section V of [3] supports the conclusion.

Moreover, there is a loophole in the logic of the "proof" in [1]. E_n in its Eq.(11) can be replaced by any number other than E_m to complete its following "proof". If a replacement is done, then nothing useful can be deduced by its "proof". The loophole arises from the suspicious " $\simeq$ " in Eq.(13) in [1]. In fact, the second term in the right side of Eq.(12) in [1], $\frac{\langle E_m|E_n|\psi\rangle}{E_m - E_n} = \frac{E_n c_m}{E_m - E_n}$, is at the same order of (or even much larger than) c_m. It can not be simply substituted by $\frac{\langle E_m|E_n|\psi^{adi}\rangle}{E_m - E_n} = 0$.

In conclusion, Ref. [1] accomplished its 'proof' by introducing an extra constraint, Eq.(6) in [1], which means their "necessary condition" can only be applied to a small class of systems. In addition there is a logical loophole in its proof. Thereby the proof is generally not reliable and their comments on others' models in other papers are unreasonable.

One direct reason why adiabatic approximation is important is it can provide an approximate but convenient approach to control complicated evolution in quantum systems. And the geometric phase developed in 1980's further widens its applications. Therefore we should only be concerned with the fidelity between the adiabatic state and evolving state, a simple and natural choice. Some special models[4] may be noted as models failing to achieve geometric phase. However, it's not proper to exclude them from being classified as adiabatic approximation processes with an extra artificial requirement (see Eq.(6) in [1]). We believe a general necessary condition should be deduced based on the only requirement that the adiabatic state should be close to the evolution state.

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